Announcements

MP5 available, due 3/31, 11:59p.

B-trees (the only “out of core” data structure we’ll discuss)

Implementation of a dictionary for BIG data.

Can we always fit data into main memory?

So where do we keep the data?

Big-O analysis assumes uniform time for all operations.

But…

The Story on Disks

2GHz machine gives around 2m instructions per _____.

Seek time around _______ for a current hard disk.

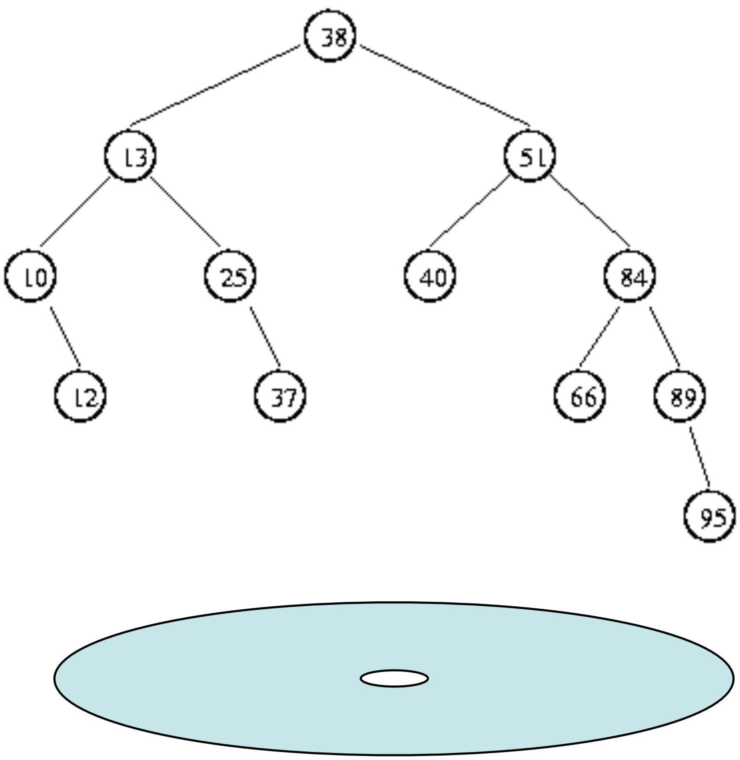
Imagine an AVL tree storing US driving records.

- How many records?
- How much data, in total?
- How deep is the AVL tree?

- How many disk seeks to find a record?

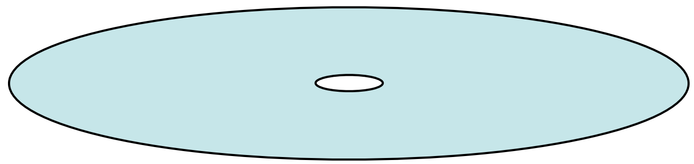
B Trees

Suppose we weren't careful…



B Tree of order m

12	18	27	52	58	63	77	89



Goal: Minimize the number of reads from disk

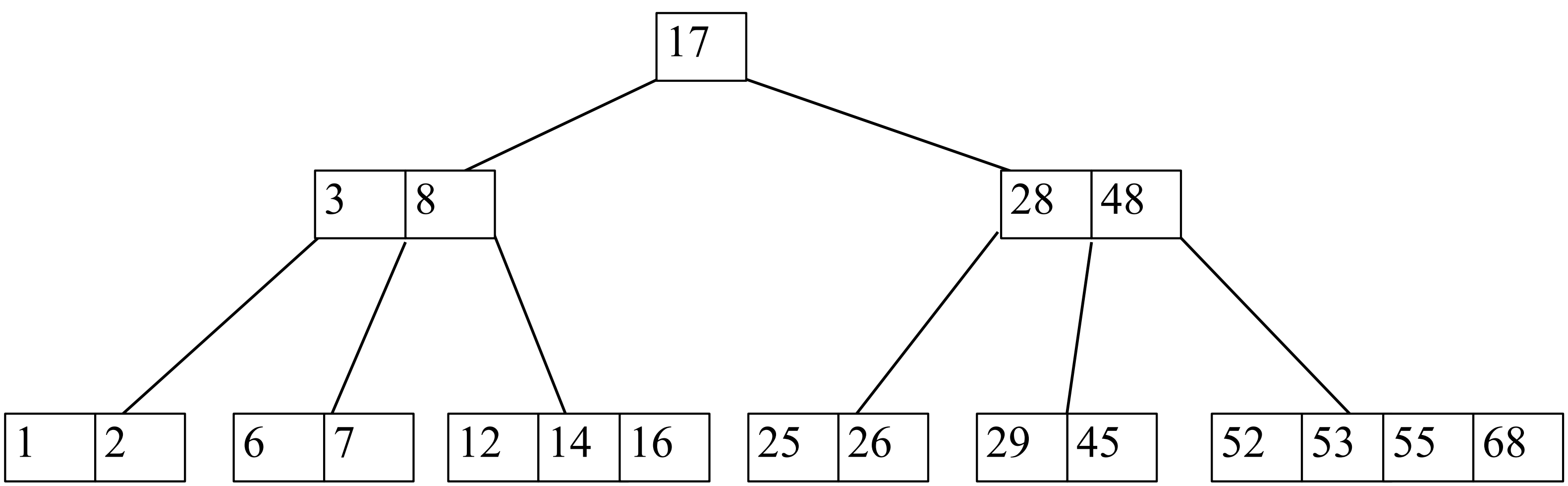
- Build a tree that uses 1 disk block per node
  - Disk block is the fundamental unit of transfer
- Nodes will have more than 1 key
- Tree should be balanced and shallow
  - In practice branching factors over 1000 often used

http://people.ksp.sk/~kuko/bak/big/

Definition of a B-tree

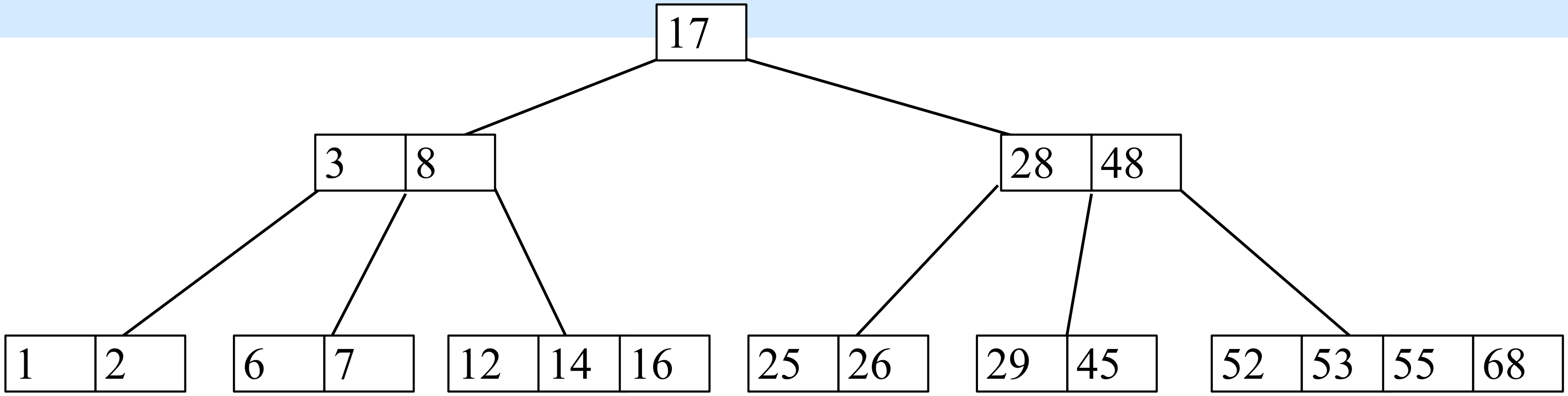
B-tree of order *m* is an *m*-way tree

- For an internal node, # keys = #children -1
- All leaves are on the same level
- All leaves hold no more than *m*-1 keys
- All non-root internal nodes have between $\lceil m/2 \rceil$ and m children
- Root can be a leaf or have between 2 and *m* children.
- Keys in a node are ordered.



Searching a B-tree

```
bool B-TREE-SEARCH(BtreeNode & x, T key){
   int i = 0;
   while ((i < x.numkeys) && (key > x.key[i]))
      i++;
   if ((i < x.numkeys) && (key == x.key[i]))
      return true;
   if (x.leaf == true)
      return false;
   else{
      BtreeNode b=DISK-READ(x.child[i]);
      return B-TREE-SEARCH(b,key);
   }
}
```



Analysis of B-Trees (order *m*)

The height of the B-tree determines the number of disk seeks possible in a search for data.

We want to be able to say that the height of the structure and thus the number of disk seeks is no more than ________.

As we saw in the case of AVL trees, finding an upper bound on the height (given n) is the same as finding a lower bound on the number of keys (given h).

We seek a relationship between the height of the structure (h) and the amount of data it contains (n).

Analysis of B-Trees (order *m*)

We seek a relationship between the height of the structure (h) and the amount of data it contains (n).

- The minimum number of *nodes* in each level of a B-tree of order *m*:
  (For your convenience, let t = _______.)

  root

  level 1

  level 2

  . . .

  level h

- The total number of nodes is the sum of these:

- So, the least **total** number of *keys* is:

Analysis of B-Trees (order *m*)

We seek a relationship between the height of the structure (h) and the amount of data it contains (n). (continued…)

- So, the least **total** number of *keys* is:

- rewrite as an inequality about n, the total number of keys:

- rewrite **that** as an inequality about h, the height of the tree (note that this bounds the number of disk seeks):

Summary

B-Tree search:

- O(m) time per node
- O(log$_m$ n) height implies O(m log$_m$ n) total time
- BUT:

Insert and Delete have similar stories.

What you should know:

- Motivation
- Definition
- Search algorithm and analysis

What you should not know:

- Insert and Delete